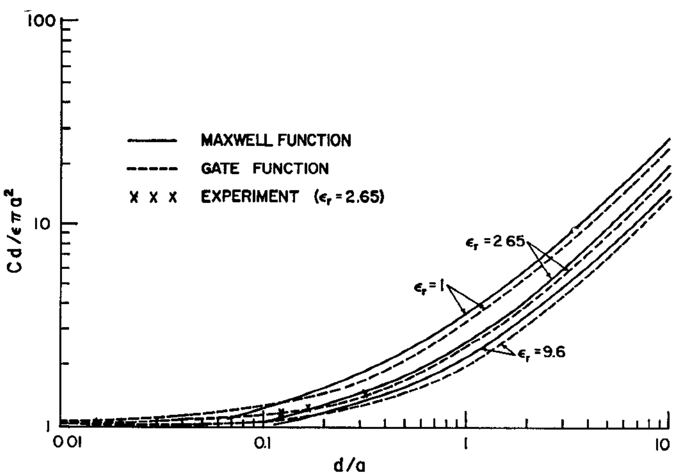

Fig. 2. Capacitance of a circular disk (normalized by $\epsilon\pi a^2/d$, $\epsilon = \epsilon_0\epsilon_r$).

$$a_m = \int_0^\infty \tilde{\rho}_m(\alpha)\tilde{\phi}_i(\alpha, d)\alpha\, d\alpha = \int_0^a \rho_m(r) r\, dr. \tag{9}$$

In the derivation of (9), Parseval's relation has been used; the use of this relation results in the elimination of $\tilde{\phi}_0$ since the inverses of $\tilde{\phi}_0$ and $\tilde{\rho}_m$ are nonzero only at the complementary regions of r.

The total capacitance of the structure is

$$C = \int_0^a 2\pi r \rho(r)\, dr = 2\pi \sum_{n=1}^{N} a_n d_n. \tag{10}$$

For the numerical calculation we have chosen $N=1$, although the accuracy of the result can be improved by increasing N. The two types of functions tested were as follows.

1) Maxwell function:

$$\rho_1(r) = \begin{cases} \dfrac{1}{\sqrt{a^2 - r^2}}, & r < a, \\ 0, & r > a. \end{cases} \qquad \tilde{\rho}_1(\alpha) = \frac{\sin \alpha a}{\alpha}$$

2) Gate function:

$$\rho_1(r) = \begin{cases} 1, & r < a, \\ 0, & r > a. \end{cases} \qquad \tilde{\rho}_1(\alpha) = \frac{aJ_1(\alpha a)}{\alpha}$$

Fig. 2 shows the capacitance of a circular disk for three different substrates, viz., $\epsilon_r = 1$, 2.65, and 9.6, calculated by using two different choices of basis functions. Note that there are crossover points for the two curves obtained by using the Maxwell function and the gate function. Since the capacitance given by (10) gives a stationary value for a trial set of basis functions, and since the one which maximizes the value of C yields a result closest to the exact one, it is evident that the Maxwell function should be used for d/a values above the crossover point, while the gate function will give more accurate results below it. The reason why the gate function gives better results for larger disks even though it ignores the edge behavior is perhaps due to the fact that the contribution of the edge singularity to the total capacitance of the large disk is a relatively small quantity. It is noted that for $d/a > 0.5$, the numerical results using the gate function are about 10 percent lower, and hence less accurate than the corresponding results for the Maxwell function.

The discrepancy between the two results is even greater for $d/a < 0.1$, where the gate function results are now more accurate.

As expected, for small values of d/a, C approaches $\epsilon_r\epsilon_0\pi a^2/d$, which is the value of the capacitance that would be obtained by neglecting the fringe effects. For $\epsilon_r = 1$, it is known that $Cd/(\epsilon\pi a^2)$ approaches $8d/(\pi a)$ as $d/a \to \infty$. For $d/a = 10$, this asymptotic value is approximately 25.5 and the numerical value computed by the present method is 26.2.

The required computation time for the above calculations with four-digit accuracy was about 6 s per structure for the choice 1) and 60 s for 2),[1] both on the CDC G-20 computer. For comparison purposes, this computer is about seven to ten times slower than the IBM 360/75.

In order to check the accuracy of the computed results, the capacitance of the actual circular disks on the substrate of $\epsilon_r = 2.65$ has been measured at 1.592 MHz. Fig. 2 shows that the experimental results are in excellent agreement with the numerical computation; in fact, the measured and the computed values differ by less than 3 percent. To conclude the discussion we might add that the principal advantage of the method is its numerical efficiency. An important feature of the method is that the numerical effort involved is not too dependent upon the physical size of the structure. In contrast, in most conventional numerical methods the computational effort is directly proportional to the size of the structure which in turn determines the size of the associated matrix.

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[1] This is because of the use of the time-consuming BESJ subroutine for the calculation of J_1. However, it is possible to reduce the computation time for J_1 by employing the polynomial approximations for J_1 given in [4].

Letters

Comments on "Analysis of Automatic Homodyne Method Amplitude and Phase Measurements"

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In the above short paper,[1] on page 623, the authors state: "Phase quadrature between the homodyne and the modulated carriers produces a null in the detector output. . . ." This is only true if the modulated carrier is completely suppressed, which is the ideal case discussed by Robertson [9].[1] Inspection of the phasor when the carrier is not suppressed, as in Schafer [11],[1] shows that the null is produced when the modulated carrier is in phase quadrature with the resultant of the homodyne and modulated carriers. The error introduced by the authors' assumption of quadrature conditions varies from less than 0.01° for a 90-dB ratio to 90° for equality of the two signals. In most applications this error is less than 0.6° (40 dB or greater ratio), and for moderate accuracies it can be ignored. For more precise measurements, however, one must use the resultant and modulated carrier in phase-quadrature analysis.

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